MY APPROACH TO HELPING WITH INTEGRATION BY SUBSTITUTION

In my experience as a tutor, one topic that often challenges first-time students in Calculus is the method of Integration by Substitution. Integration, which is often taught at the end of the semester is sometimes hurried, and many students have trouble getting used to Calculus being more than just taking derivatives. For some students it goes against their intuition, as they do not have much experience evaluating integrals. I introduce the following ideas to any student that needs help with "u substitution." Here is one example that a student and I might work through to get a better understanding of the technique:

$$\int x^2 sin(x^3)dx$$

Students do not have a method to evaluate this aside from substitution. After explaining to students that these methods are the integrals of "chain rule" derivatives and that we are making the integral readily solvable by removing extra terms that are being multiplied (the x^2), we begin. Firstly, I recommend students set the substitution variable u to the term that is inside parenthesis, a radical, or the exponent. For this example:

$$u = x^3$$

The first major difference between the common approach and my own occurs in this step, especially when working on a first example. I suggest immediately plugging in u. Thus,

$$\int x^2 sin(u)dx$$

Next, I make it clear to the student that we have a u in our integral and x as our variable of integration and we need to make u our variable of integration. That is, we need du in the integrand not dx. We refer to our statement of u to derive du, here:

$$\frac{du}{dx} = 3x^2$$

The biggest difference between methods occurs in this step, as we do not want dx we will isolate it and substitute the result into the integral. We now have,

$$\frac{du}{3x^2} = dx$$

$$\int x^2 sin(u) \left(\frac{du}{3x^2} \right)$$

Rather than manipulating our differentiated equation to match terms in the integrand exactly, we isolated the dx and now we can cancel the x^2's. This method is generally the same sequence of steps every time, isolating then canceling, rather than a unique manipulation for each problem. This complication frequently confuses students when they are already having difficulty with integrals. Let us now finish our problem, we are currently at:

$$\frac{1}{3} \int sin(u) du$$

Most students should be able to determine this anti-derivative if they recall the derivative of $-cos(x)$ is $sin(x)$. Therefore,

$$\frac{1}{3} \int sin(u) du = -\frac{1}{3} cos(u)$$

Lastly, a reminder to students is that if the problem starts in terms of x we want to end in terms of x. We finish the integral by plugging the u equation into our solution. Thus,

$$\int x^2 sin(x^3) dx = -\frac{1}{3} cos(x^3)$$

The student and I may work through one or two examples together before he or she feels comfortable working through one by themselves. As they work through the problem I will mention the key steps if necessary. The main points to say if they get stuck include: we want to define u, take the derivative of u, solve for dx, plug in and cancel, integrate, change back to x. Once the student has completed a few exercises by this process, he or she feels more confident about working on "simple" integrals and "u substitution" problems. A final word to students is that if you can see that the derivative of the term in parenthesis and the multiplied term are similar, you will have selected the correct u.