Michael’s Awesome Derivative Rules Sheet

This sheet lists and explains many of the rules used (in Calculus 1) to take the derivative of many types of functions.

### General Handy Rules

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>The derivative of any constant number (2, -2.544, 200.8) is zero</td>
<td>[ \frac{d}{dx} c = 0 ] Example: [ \frac{d}{dx} 4.6 = 0 ]</td>
</tr>
<tr>
<td>The Product Rule</td>
<td>[ \frac{d}{dx} f(x)g(x) = f’(x)g(x) + g’(x)f(x) ] Example: [ \frac{d}{dx} xe^x = e^x + xe^x ]</td>
</tr>
<tr>
<td>The Power Rule</td>
<td>[ \frac{d}{dx} c \cdot x^n = c \cdot n \cdot x^{n-1} ] Where ( x ) is a variable. Where ( c ) and ( n ) are just numbers. Example: [ \frac{d}{dx} 3x^2 = 3 \cdot 2 \cdot x^{2-1} ]</td>
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<tr>
<td>The Quotient Rule</td>
<td>[ \frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f’(x)g(x) - g’(x)f(x)}{g(x)^2} ] Example: [ \frac{d}{dx} \frac{1}{x} = \frac{-1}{x^2} ]</td>
</tr>
<tr>
<td>Derivative of ( e^x ) or ( e ) to the power of any function</td>
<td>[ \frac{d}{dx} e^x = e^x ] Example: [ \frac{d}{dx} e^{2x} = 2 \cdot e^{2x} ]</td>
</tr>
<tr>
<td>Derivative of ( \ln(x) ) or any function inside the ( \ln )</td>
<td>[ \frac{d}{dx} \ln(f(x)) = \frac{1}{f(x)} \cdot f’(x) ] Example: [ \frac{d}{dx} \ln(4x) = \frac{1}{4x} \cdot 4 ]</td>
</tr>
</tbody>
</table>

### Helpful Tips:

1. **Implicit differentiation**: When you have a function that contains two variables (x and y). Remember that when you take the derivative of y, you end up with a \( \frac{dy}{dx} \).
# Trig Table

<table>
<thead>
<tr>
<th>Trig function</th>
<th>Corresponding derivative function</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d}{dx}\sin(x) = \cos(x)$</td>
<td>$\frac{d}{dx}\cos(x) = -\sin(x)$</td>
<td>$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$</td>
<td>$\frac{d}{dx}\tan(x) = \sec^2(x)$</td>
<td>$\frac{d}{dx}\cot(x) = -\csc^2(x)$</td>
<td>$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$</td>
</tr>
</tbody>
</table>

## Taking the Derivative of a trig function:

A trig function has an inner function:

$$\sin(8x^2-3)$$

**Step 1:** Locate the derivative function of the trig function. See the above table for reference.

**Step 2:** Put the inner function that is inside the original trig function inside the derivative function.

**Last Step:** Multiply the derivative function by the derivative of the inner function.

For example:

$$\frac{d}{dx}\sin(3x^2) = \cos(3x^2) \cdot 3 \cdot 2 \cdot x$$

Following the Steps:

1. $\cos$ is the derivative function of $\sin$
2. $3x^2$ is the inner function inside the $\sin$ function. So we put $3x^2$ inside the $\cos$ function.
3. $3 \cdot 2 \cdot x$ or $6x$ is the derivative of $3x^2$ so we multiply it to the $\cos$ function.

## Chain Rule: (a function within a function)

**We take the derivative of the outer function first then work our way inside**

1. **(with trig or ln functions)**

   $$\frac{d}{dx} 2\sin^3(7x) = 6\sin^2(7x) \cdot \cos(7x) \cdot 7$$

   a) **We apply the power to the $\sin$ function.**

   the new coefficient (6) is obtained by multiplying the old coefficient (2) and the old exponent (3).

   the new coefficient is then multiplied by the same function (which is the $\sin$ function) raised to the same power (3) minus 1. $\rightarrow (\sin(7x))^2$

   b) We multiply that by the derivative of the $\sin(7x)$ (See Taking the Derivative of a trig function).

2. **(without trig or ln functions)**

   $$\frac{d}{dx} 2(3x+8)^3 = 6(3x+8)^2 \cdot 3$$

   a) **We apply the power rule to the $3x+8$ function.**

   the new coefficient (6) is obtained by multiplying the old coefficient (2) and the old exponent (3).

   the new coefficient is then multiplied by the same function (which is the $(3x+8)$ function) raised to the same power (3) minus 1. $\rightarrow (3x+8)^2$

   b) We multiply that by the derivative of $(3x+8)$